Vortices in superconductors

1999 J. Phys.: Condens. Matter 11 7685

(http://iopscience.iop.org/0953-8984/11/40/302)

J. Phys.: Condens. Matter **11** (1999) 7685–7694. Printed in the UK PII: S0953-8984(99)03125-2

Vortices in superconductors

E M Forgan
School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, UK

Received 31 March 1999

Abstract. A personal view is given of important events in the research led by Joe Vinen on vortices in superconductors. The effect of electron–hole ('Andreev') scattering on transport processes, such as sound attenuation and thermal conductivity, in the mixed state is described. Theoretical work by Nozieres and Vinen on flux flow is shown still to be topical. The role of fluctuations in the transition to the normal state from the mixed state is described. These early experiments are reviewed in the light of evolving ideas about flux lattice melting which have been generated by high-T_c research. Finally, recent observations of flux lines, using neutron and muon techniques, are linked to the work inspired by Joe Vinen. An appreciation is given of his distinctive approach to physics.

1. Beginnings

In this paper are included some selected historical items, combined with modern developments in the field which has been revived by the advent of high-T_c superconductors. I first met Joe Vinen when I came to Birmingham to be interviewed by him for a research studentship in his group. I distinctly remember being impressed because he *said very little*, and handed me a theoretical paper about flux lines written in French. He gave me a word of advice to help in interpreting it: to remember that the French word 'diffusion' means 'scattering'. Impressed by his taciturnity and his misplaced belief that I might understand the paper with that help alone, I decided to accept his offer to work on flux lines in superconductors. I was supervised by Colin Gough, who is editing these contributions. It is a tribute to Joe Vinen's leadership that Colin and I have both found it most attractive to continue in Birmingham for our working lives since then. I soon became aware that Joe's quiet demeanour covered a *huge* understanding of physics. When Joe hesitates, it means that he realizes the ramifications of a question that we all *thought* we understood. Chris Muirhead relates that Joe came in one morning and said: 'I don't think I understand Ohm's law'. However, we soon discovered that Joe's 'ignorance' was rather deeper than our knowledge!

2. Transport processes in the mixed state

I turn from this brief and partial character sketch to the physics of flux lines. I will describe a discovery that Colin Gough and I stumbled upon in my first year as a research student here. I was assigned to a project on the absorption of ultrasonic sound waves in superconductors. This appears to be a rather abstruse kind of measurement, but is actually rather important because it allows one to detect the 'normal' electrons below T_c. If one measures the *resistance* of a superconductor, it drops to zero at T_c as soon as a tiny fraction of the electrons have condensed into Cooper pairs: the superconductivity of these few electrons 'shorts out' the

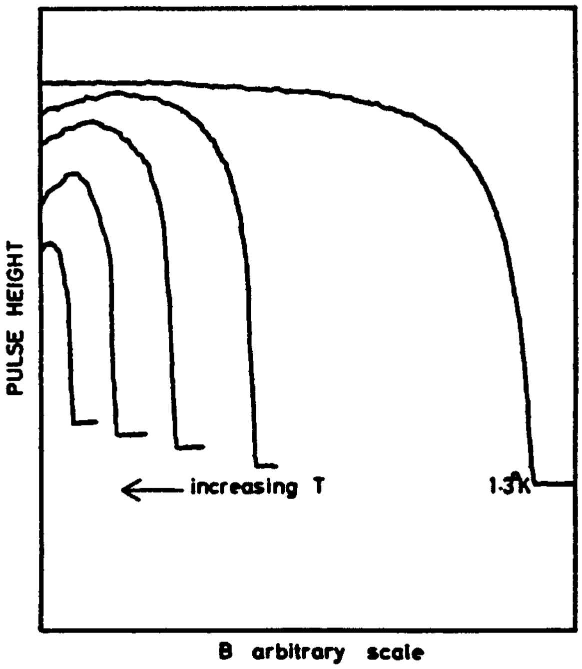

Figure 1. Height of the first echo of a pulse of longitudinal 30 MHz sound reflected along a rod of pure niobium, as a function of the average induction inside the sample at various fixed temperatures below T_c. Reprinted from [3], © 1966, with permission from Elsevier Science.

uncondensed electrons, and makes them invisible. However, the normal electrons still respond to the distortion of the lattice in a sound wave, so the measurement of sound absorption below T_c allows the properties of the normal electron fraction to be measured over the whole temperature range. Indeed, measurements of the temperature-dependence of ultrasonic absorption in a superconductor [1] had been an important confirmation of the Bardeen–Cooper–Schrieffer (BCS) theory of superconductivity [2] which had appeared some years earlier.

One of our main aims at that time was to establish a detailed understanding of ‘type II’ superconductors. These are materials in which the application of a magnetic field causes a *gradual* destruction of superconductivity, by the entry of quantized magnetic flux lines to form the *mixed state*. At a sufficiently high value of applied field, the ‘normal’ cores of the flux lines are close together and strongly overlap, so that superconductivity disappears at the ‘upper critical field’ H_{c2}. In the mixed state, each flux line is surrounded by a vortex of flowing supercurrent, somewhat analogous to the vortex lines in superfluid helium that Joe had done so much to investigate.

Hence, we measured the attenuation of sound through the mixed state of ultrapure niobium—one of the few elements which is a type II superconductor, with a T_c of 9.28 K. Typical results are shown in figure 1. At low temperatures the absorption followed simple expectations: at zero field the attenuation was small and the height of the sound echo correspondingly large; as superconductivity was destroyed by the application of a magnetic field, the absorption increased until the normal state was reached at H_{c2}. At higher temperatures, there was a noticeable normal fraction which caused some sound attenuation at zero field; however, the echo height did *not* decrease monotonically as the field was increased to H_{c2}. Instead, there was a *decrease* in attenuation as the flux lines first entered the specimen.

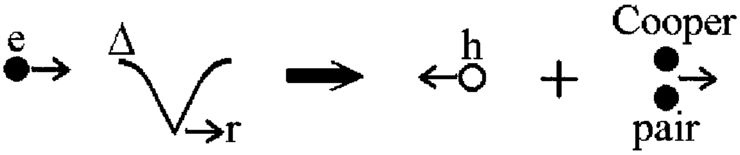

Figure 2. Andreev scattering: an electron-like quasi-particle is scattered by a spatial modulation of the gap parameter and becomes a hole-like quasi-particle. The process is elastic, so the energy of the hole is the same as that of the electron. The hole has nearly the same momentum as the electron, as expected, but its *group velocity* is quite different. Electrical charge and current are conserved by the extra Cooper pair in the condensate.

Under the conditions of our experiment, sound absorption is proportional to electron mean free path, just like electrical conductivity. (This effect can be seen in the normal state: the attenuation decreases as the temperature increases because the phonon effects on electron mean free path are noticeable in this high-purity sample.) It seemed most unlikely that the normal fraction was decreasing as the flux lines entered. We therefore concluded that the reduction in attenuation arose because of a reduction in the mean free path of the normal electrons, as they were scattered by the flux line cores. This conclusion was supported by estimates of the scattering cross section of the flux lines from our results. We obtained values comparable with the core diameter, which seemed very reasonable.

These results were published in a letter [3] which also included some delightful misprints: EMF acknowledged a studentship from the Scientific Research Council (when has the funding of our research ever been *scientific*?!), and both authors expressed their gratefulness to Professor W F Vinen for making *helful* suggestions. After the letter was published, a helpful suggestion came from Joe, expressing a fundamental worry about our interpretation. His argument was quite simple: flux line cores are *large* (typically hundreds of angstroms in niobium), which is much bigger than the Fermi wavelength of the electrons. Hence flux lines *cannot* alter the momentum of the electrons by a large amount. Now in the case of electrical conductivity, it is well known that *large angle scattering* (with a large change in electron momentum) is necessary for a scattering event to have a big effect on the effective mean free path. Why should this not also be the case in sound absorption which involves very similar physics: in effect the *viscosity* of the electrons?

Typically, Joe not only provided the question before we had thought of it, but also the answer. It lay in a process which is usually known now as *Andreev scattering* [4] and is illustrated in figure 2. This process is the scattering of an electron-like excitation into a hole-like state (or *vice versa*). It can only occur in a superconductor, and arises because of the mixing of electron and hole states near the Fermi surface as given by the BCS theory. The scattering is caused by the spatial variation of the gap parameter, which goes to zero along the axis of a flux line at the middle of the core. The apparent non-conservation of charge in the electron–hole scattering is taken care of by the large and indefinite number of Cooper pairs in the condensate. Andreev scattering affects the ultrasonic attenuation, *not* because it alters the quasi-particle *momentum* noticeably, but because it alters the *velocity* by a large amount. When one looks in detail at the distribution functions of the excitations [5], it is clear that Andreev scattering relaxes these distributions and effectively reduces the carrier mean free path. This scattering is also effective in reducing thermal conductivity, as was experimentally demonstrated by Joe and collaborators [5], but not in electrical conductivity. Similar results for ultrasonic attenuation and thermal conductivity were obtained by other authors [6, 7] and detailed calculations of the scattering cross section of a flux line were carried out by Cleary [8].

As a result of this research we understood a little more about the fickle concept of mean free path and how important it is to be self-critical, even after you believe that you have understood something. In all of this, Joe was our guide.

I now turn to two theory papers from this period: the first is the only paper I have been sole co-author with Joe, on the theory of ultrasound absorption in a superconductor [9]. The work was at least 90% Joe's, but with typical generosity (or careful adherence to alphabetic ordering of surnames, which amounted to the same thing with a surname like Vinen) he put me as first author. I only mention this paper, because in it Joe departed from his incredibly high standards, just for once. In a footnote we say: 'For simplicity we omit a factor of $2/(2\pi)^3$ from all integrals'. In other words, *for simplicity* we have redefined 124.02 to be equal to 1: rather more severe than the US state which is supposed to have set π equal to 3 for similar reasons! In fact, whether present or not, the aforementioned factor cancelled in all the final results, which showed that despite their complex electron–hole nature, the excited quasiparticles in a superconductor could be treated by the standard Boltzmann transport equation, just like electrons in metals or excitations in liquid helium. The second paper, by Nozieres and Vinen [10] addressed the *motion* of flux lines in superconductors. This was and remains of fundamental and practical interest: when a transport current flows through a superconductor in the mixed state, it causes a sideways force on the flux lines, which can cause them to move and dissipate energy despite the presence of superconductivity. Nozieres and Vinen showed that the sideways force is analogous to the *Magnus force* which acts on any spinning object (such as a tennis ball) as a fluid moves past it. (Of course, Joe made analogies with similar effects in liquid helium.) They considered in detail how this force is transmitted to the flux line and balanced by a frictional force between the flux line core and the crystal lattice. Another question which they addressed (and in more precise terms than a well known paper by Bardeen and Stephen [11]) was whether the flux line motion is *exactly* perpendicular to the current. In fact the motion is not exactly perpendicular, and this corresponds to the Hall effect. Now to my present knowledge, there is no agreement on the explanation of observations of the Hall effect in high-T_c superconductors [12]. Hence, this paper from 1966 has been rediscovered; it remains of current interest and is frequently cited in the current literature.

If the motion of flux lines is of fundamental interest, then the *pinning* of flux lines to prevent that motion is of great practical interest, since this is the only way of achieving resistanceless current flow at high fields. The science of pinning has advanced hugely since the 1960s, not only in terms of the concepts (see for example [13]) but also because vortices in high-T_c materials have proved much more difficult to pin at elevated temperatures. It may be that to use these materials at high fields, we may still be forced to use liquid helium temperatures.

3. The phase transition from mixed to normal state

Because of their generally long coherence length, conventional superconductors have always been regarded as a prime example of systems that show a simple mean-field phase transition, with critical fluctuations playing a role only in a very small temperature region near T_c. However, Joe's experience with helium which has a very short coherence length, and is dominated by fluctuations at T_λ, encouraged him to search for similar effects in superconductors. These were difficult experiments, and I well remember a search for fluctuation diamagnetism in lead, which was the last time I saw a galvanometer amplifier used in earnest. However, these experiments finally bore fruit in Colin Gough's observation of the transition from the mixed state to the normal state. The overall picture of this transition is in figure 3, which shows some heat capacity measurements taken with a coarse temperature scale. At zero applied field there is a jump in heat capacity at T_c, just as expected from mean field

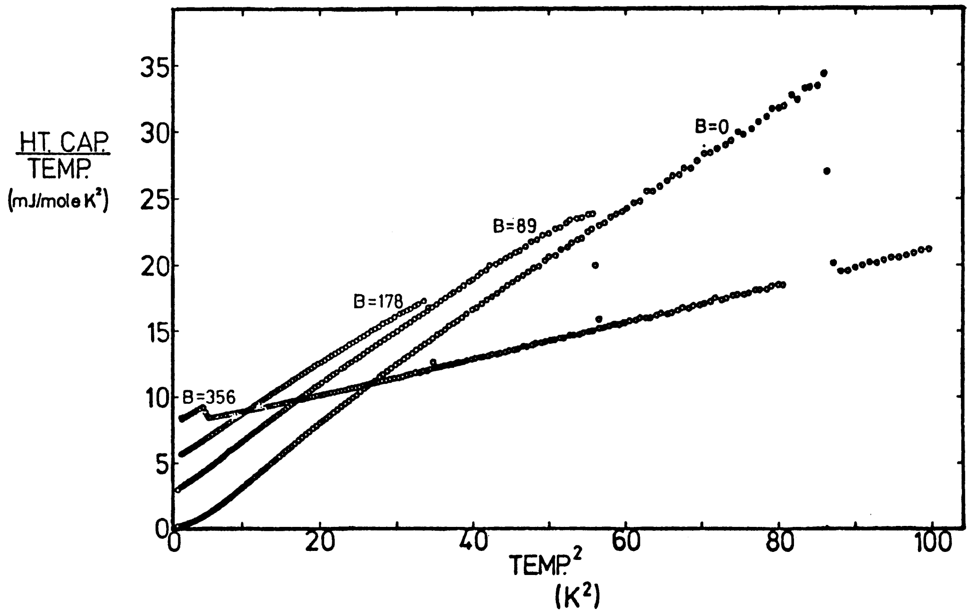

Figure 3. Heat capacity versus temperature for niobium at various fixed magnetic inductions (values given in mT). Reprinted from [14], © 1981, with permission from Elsevier Science.

theory for a second-order phase transition. Similarly, at non-zero induction a second-order phase transition is seen, but now at a lower temperature where the H_{c2} line is crossed. In this case, the transition is from the normal state into the mixed state. (Incidentally, one notices at low temperatures the heat capacity from the normal cores of the flux lines in the mixed state—but that is another story.) However, if we look closely at the transition in a field the transition is clearly *not sharp*. As shown in figure 4, it is smeared over a temperature interval of the order of millikelvin [15]. This is *not* due to sample inhomogeneity (the experiment was performed on probably the best niobium crystal in the world, grown in Birmingham) but due to critical fluctuations which are much enhanced in the mixed state. This was shown in the theory [16] by David Thouless, who was at Birmingham at the time.

There is a consequence which flows from these observations, the importance of which has only become clear with the advent of high-T_c superconductors. The *smearing* of the transition means that there is no true phase transition at H_{c2}, only a 'crossover' where the properties change rapidly. Hence, the mixed state differs only in *degree* from the normal state: it is not a new state. This means that the electrical resistivity in the mixed state does not differ qualitatively from that in the normal state, and can never be *precisely zero*, unless there is another phase transition. What we never looked for at the time was a true phase transition to a zero-resistance state. In recent years, such transitions have been proposed for high-T_c materials, with the low-temperature, zero-resistance phase proposed to be a 'vortex glass' in the presence of strong disorder [13]. If the disorder is weak, then current ideas suggest that a somewhat disordered flux line lattice (known as a 'Bragg glass' [17]) is formed, and that this undergoes *flux lattice melting* into a state continuous with the normal state. It certainly seems reasonable that if a flux lattice is formed, then it should melt like any other lattice, and indeed the formation of a fluid mixed state was proposed long before high-T_c superconductors [18]. It should be mentioned that a few voices [19] have claimed that the flux line lattice

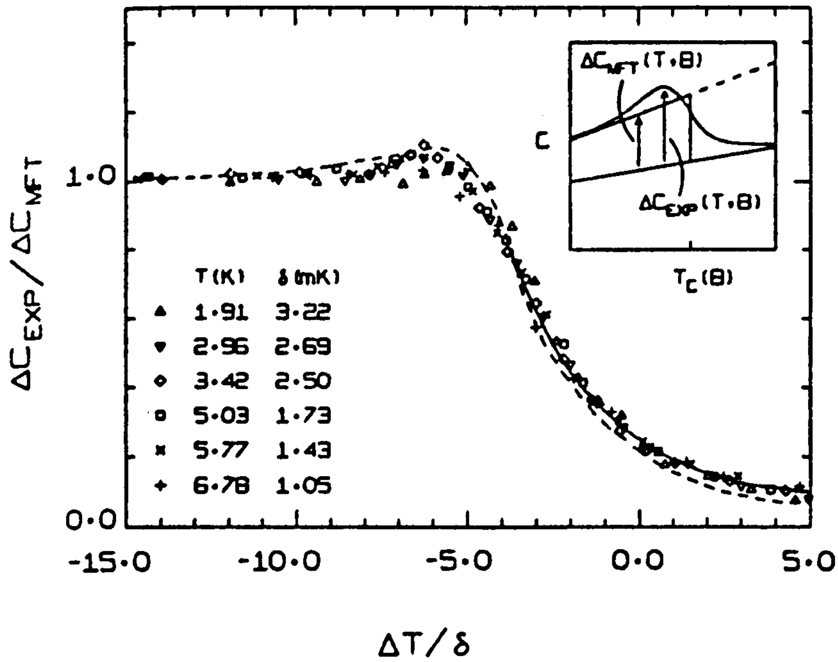

Figure 4. Fluctuation heat capacity of ultrapure niobium near H_{c2}. The data at various values of magnetic induction are replotted in scaled form, as given by theory [16]. The vertical axis is expressed as a fraction of the mean field heat capacity jump and the temperature axis is scaled in units of a quantity δ, which has values of the order of millikelvin. Reprinted with permission from [15], © 1975, the American Physical Society.

never melts because it never crystallizes, and the observations shown in figure 4 represent *all* that is happening. What is certainly true is that high-T_c superconductors are much more like liquid helium than conventional superconductors, and the stability of flux lines with respect to thermal fluctuations is reduced in these materials by their strongly layered nature, their large κ, and their high critical temperatures. As a result, zero resistance certainly does not extend all the way from zero field to the mean-field H_{c2} [13].

4. Recent observations of flux lines in high-T_c and other materials

Perhaps because of ‘the nail in the coffin of superconductivity’ [20], many of us at Birmingham left that subject, although I continued to study superconducting transition metal dichalcogenides for a while. These layered materials could be ‘intercalated’ with organic molecules between the layers, and might have represented a realization of Little’s proposed structure for a high-T_c material [21] with very high-energy pairing interactions. However, the T_c stayed obstinately low, and even *fell* when some organic materials were added; nevertheless, the experience of superconductivity in layered compounds proved useful when actual high-T_c materials came along. So I moved into neutron scattering, and Joe returned to helium, using the experimental techniques of light scattering and then ion motion in the liquid: these subjects are covered in other papers in this issue. Only Colin Gough kept the superconductivity flame alight.

That flame burst out anew under Colin’s leadership when high-T_c materials passed the

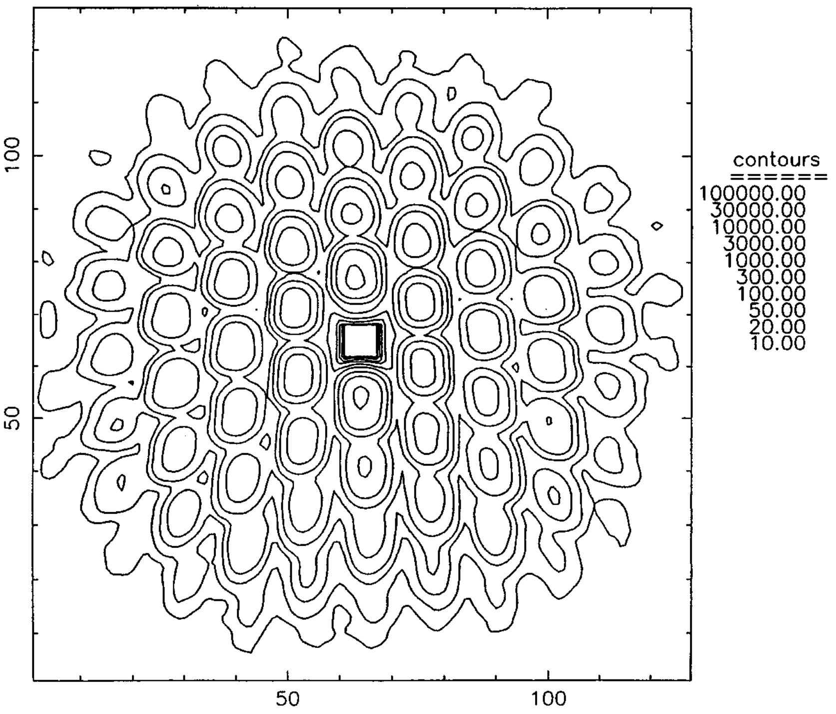

Figure 5. Diffraction pattern from flux lines in single crystal Nb, taken with long wavelength neutrons incident parallel to a field of 0.1 T; the contours are on a logarithmic scale to make the higher-order diffraction spots more visible. The spot due to the incident beam at the centre has been masked. The axes are the pixel coordinates of a neutron multidetector.

liquid nitrogen barrier in 1987, and the group was rapidly able to make a contribution in measuring the flux quantum and observing superconducting quantum interference device (SQUID) behaviour [22, 23]. I felt I might be able to combine my old interests and new ones, so attempted to observe flux line structures in a high-T_c material YBCO by small-angle neutron scattering. We began our experiments by checking that we could detect the flux lattice in niobium (one of the single crystal samples we had kept carefully from the early days of the Birmingham type-II superconductivity work). It gave a beautiful diffraction pattern with a triangular symmetry, as shown in figure 5. The experiment on YBCO was not so easy. The longer penetration depth made the flux line signal *much* weaker, and defects in the material gave a substantial background scattering that had to be subtracted to show the scattering by the flux lines. Our collaboration succeeded at the second attempt, and obtained the weak pattern shown in figure 6. In reporting the results [24], we showed how *sometimes* it is possible to burrow below the Poisson noise and see details that are scarcely visible. Of figure 6, we wrote: 'Bragg spots . . . can be detected in positions that are aligned with the twinned crystalline axes in an average fourfold symmetry'; this conclusion was only confirmed when we got better at the experiments and obtained data such as that shown in figure 7.

Another important development in this subject was the observation of the flux lattice in the very anisotropic high-T_c compound BSCCO [25, 26] using neutrons and muons. Unlike in YBCO, the flux lattice signal disappeared well below T_c, which we interpret as a *melting* of the flux lattice. Other groups observed jumps in resistivity [27], magnetization [28] and a latent heat [29], all of which have been identified with the melting transition. In addition

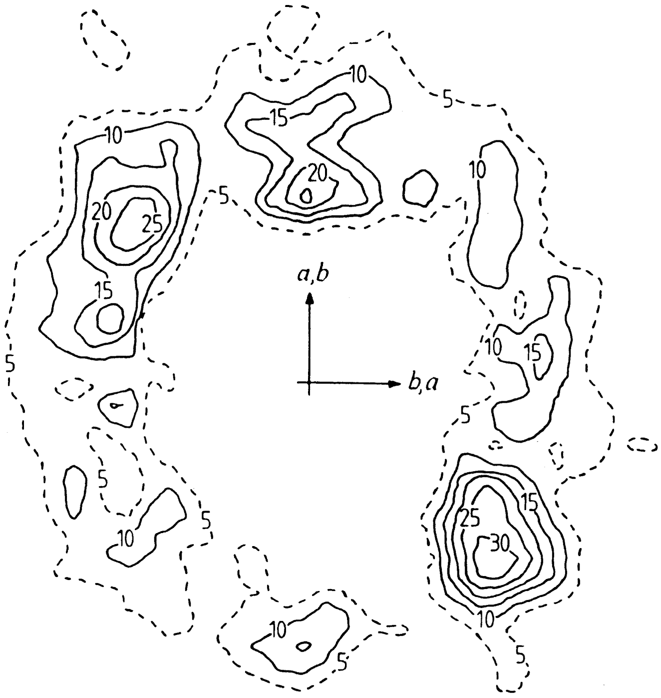

Figure 6. The first ever neutron diffraction pattern from flux lines in YBCO, taken with a field of 0.2 T approximately parallel to the c-axis. Reprinted with permission from [24], © 1990, Macmillan Magazines Limited.

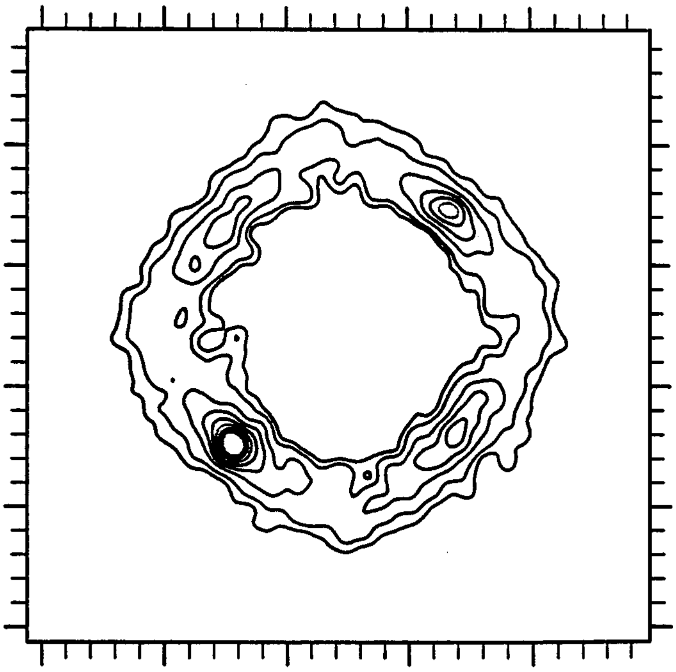

Figure 7. A better neutron diffraction pattern from flux lines in YBCO, taken with a field of 0.2 T parallel to the c-axis, showing a fourfold structure due to pinning of flux lines to twin planes.

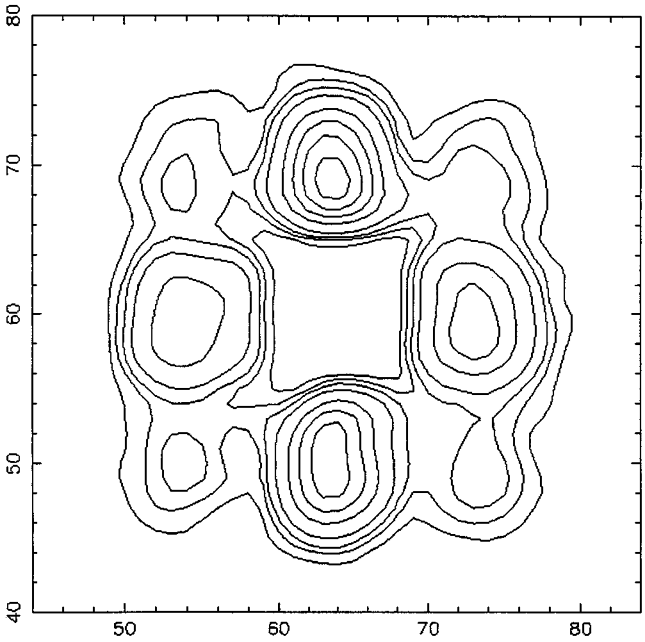

Figure 8. A neutron diffraction pattern obtained from flux lines in Sr_2RuO_4, taken with a field of 20 mT parallel to the c-axis, showing a square lattice due to p-wave pairing and the Fermi surface anisotropy (see [32]).

to being destroyed by melting, the flux lattice signal also disappeared when the field was increased to $\lesssim 0.1$ T—a field *much* smaller than B_{c2}. It appears that this is a sign of flux lines decomposing into 'pancake vortices', which are whirlpools of supercurrent located in individual superconducting planes. Such research has continued, with further information acquired about the pinning, melting and anisotropic behaviour of flux lines, and has even extended down to low-T_c materials to use new neutron techniques for investigation of *flux line motion* under the influence of a current [30, 31].

A final example of a combination of the old interest in type-II superconductors, fired by Joe, and new materials and techniques, is the observation of a *square* flux lattice in strontium ruthenate (figure 8). This material is isostructural with the first of the high-T_c materials discovered by Bednorz and Müller, but has a T_c of less than 1.5 K and a very low critical field, so there is no possibility of it being *commercially* useful. However, it appears that the electron pairing in this material is *p-wave*, like superfluid ^{3}He. By a series of detailed arguments given elsewhere [32], one can show that the square structure and orientation of the flux lattice, and the values of penetration depth and coherence length derived from the measurements are fully consistent with p-wave pairing. In addition, one can deduce the remarkable result that only half the electrons are strongly paired under the conditions of the experiment. It is clear that there are still new and exciting superconductors to be discovered, after nearly a century of investigation of the phenomenon.

5. Conclusion

I have only one thought to add: perhaps the greatest influence that Joe Vinen has had on this group has been his *example* of being an innovative but careful scientist, who has never been satisfied with slick explanations. He always aims at a thorough understanding, often backed up by theory which he developed himself. We may not always have followed this example, but nonetheless we have always had it in front of us, well described by the words of Francis Drake:

‘O Lord God, when you give to your servants to endeavour any great matter, grant us also to know that it is not the beginning but the continuing of the same until it be thoroughly finished which yields the true glory.’

References

[1] Morse R W and Bohm H V 1957 *Phys. Rev.* **108** 1094–6
[2] Bardeen J, Cooper L N and Schrieffer J R 1957 *Phys. Rev.* **108** 1175–204
[3] Forgan E M and Gough C E 1966 *Phys. Lett.* **21** 133–5
[4] Andreev A F 1964 *Sov. Phys.–JETP* **19** 1228–31
[5] Vinen W F, Forgan E M, Gough C E and Hood M J 1971 *Physica* **55** 94–113
[6] Sinclair A C E and Leibowitz J R 1968 *Phys. Rev.* **175** 596–8
[7] Lowell J and Sousa J B 1967 *Phys. Lett.* A **25** 469–70
[8] Cleary R M 1968 *Phys. Rev.* **175** 587–96
[9] Forgan E M and Vinen W F 1970 *J. Phys. C: Solid State Phys.* **3** 222–34
[10] Nozieres P and Vinen W F 1966 *Phil. Mag.* **14** 667–88
[11] Bardeen J and Stephen M J 1965 *Phys. Rev.* **140** A1197–207
[12] Brandt E H 1995 *Rep. Prog. Phys.* **58** 1465–594
[13] Blatter G *et al* 1994 *Rev. Mod. Phys.* **66** 1125–388
[14] Baker C G B, Forgan E M and Gough C E 1981 *Physica* B **108** 927–8
[15] Farrant S P and Gough C E 1975 *Phys. Rev. Lett.* **34** 943–6
[16] Thouless D J 1975 *Phys. Rev. Lett.* **34** 946–9
[17] Giamarchi T and Le Doussal P 1995 *Phys. Rev.* B **52** 1242–70
Giamarchi T and Le Doussal P 1997 *Phys. Rev.* B **55** 6577–83
[18] Eilenberger G 1967 *Phys. Rev.* **164** 628–35
[19] Moore M A 1997 *Phys. Rev.* B **55** 14 136–9
[20] Parks R D (ed) 1969 *Superconductivity* (New York: Marcel Dekker)
[21] Little W A 1964 *Phys. Rev.* A **134** 1416–24
[22] Gough C E *et al* 1987 *Nature* **326** 855
[23] Colclough M S *et al* 1987 *Nature* **328** 47–8
[24] Forgan E M *et al* 1990 *Nature* **343** 735–7
[25] Cubitt R *et al* 1993 *Nature* **365** 407–11
[26] Lee S L *et al* 1993 *Phys. Rev. Lett.* **71** 3862–5
[27] Safar H *et al* 1992 *Phys. Rev. Lett.* **69** 824–7
[28] Zeldov E *et al* 1995 *Nature* **375** 373–6
[29] Schilling A *et al* 1997 *Phys. Rev. Lett.* **78** 4833–6
[30] Forgan E M 1998 *Neutron Scattering in Layered Copper Oxide Superconductors* ed A Furrer (Dordrecht: Kluwer) pp 375–401
[31] Forgan E M *et al* 1999 *Physica* B **267/8** 115–21
[32] Riseman T M *et al* 1998 *Nature* **396** 242–5